

# Delay-Dependent Approach to Robust $H_\infty$ control and Stabilization Analysis for Uncertain Takagi-Seguno Fuzzy Systems with State and Input Time-Delays

BOURAHALA Fayçal

KHABER Farid

QUERE Laboratory, Electrotechnic Department, University Setif 1 (19000)

Bourahala1981@yahoo.fr

jfkhaber@yahoo.fr

**Abstract:**

This paper presents the robust  $H_\infty$  control problems for a class of uncertain Takagi-Sugeno (T-S) fuzzy systems with time-delay where uncertainties involved in the state and input matrices. The T-S fuzzy model is employed to represent uncertain delayed nonlinear systems. A delay-dependent stabilization criterion is first presented and new conditions of stabilization for uncertain T-S fuzzy model with time-delay are given in the form of linear matrix inequalities (LMI) and based on Lyapunov-Krasovskii functional approach. The fuzzy feedback robust  $H_\infty$  controller is designed to stabilize the T-S fuzzy system to achieve the prescribed disturbance attenuation level. Numerical example is presented to demonstrate the effectiveness of the proposed method.

**Keywords:**

T-S fuzzy model, time-delay systems, robust stability, LMI, uncertain systems, delay-dependent conditions.

## 1 Introduction

In recent years, the nonlinear systems in the form of Takagi and Sugeno fuzzy model has been extensively investigated due to its effectiveness in control system [1]. This fuzzy model has shown its advantages in using a small number of fuzzy rules for modeling the higher-order nonlinear systems. It is described by fuzzy IF-THEN rules which represent local linear input-output relations in various operating points of the nonlinear systems. The overall T-S fuzzy model is obtained by interconnecting of all subsystems through the fuzzy membership functions.

The stability and stabilization problem of T-S fuzzy model based on the candidate Lyapunov function has been extensively studied over the past two decades [2-3] and references therein,. By the parallel distributed compensation (PDC) technique the state feedback stabilization conditions can be obtained and expressed in terms of the feasibility of a set of LMI which can be solved numerically and

effectively using convex programming techniques [4].

In the original T-S fuzzy model formulation, there is no delay in the control input, output and state. However, time delays often appear in industrial systems and information networks such as chemical processes, metallurgical processing systems, network systems and long transmission lines in pneumatic, hydraulic and so on. Hence, the existence of time-delays usually becomes the source of instability and deteriorated performance of systems. Thus, it is also important to develop system theory and extend the stability and stabilization issues to nonlinear time-delay systems. Therefore, The study of delay systems has been the subject of many studies in automatic in recent years [5-7], in which some LMI conditions for stability and stabilization have been proposed and based on Lyapunov-Krasovskii functional method.

It is clear that the stability analysis and stabilization are important issues in analysis and design of fuzzy control systems with time delay. In general, there are two ways for the stability analysis and control synthesis of T-S fuzzy model with time-delay, they are delay-independent and delay-dependent approaches. For both approaches, they have their own advantages on dealing of T-S fuzzy models with time-delay. Much attention has been paid to the study of delay-dependent stability and stabilization for time-delay systems because delay-dependent results for time-delay systems are less conservative than those for the delay-independent cases, especially for time-delay systems with actually small delay. Delay-independent conditions that are independent of the size of the time delay have been proposed in [8-9], where as delay-dependent conditions have been obtained in [10-13].

In this paper, we present some sufficient conditions for the solvability of the problem of delay-dependent stability and stabilization for T-S fuzzy systems with state and input delay with parameter uncertainties.

Here we give generalized delay-dependent sufficient conditions for stability of fuzzy time-delay systems. In fact, our generalized conditions guarantee the stability of a wider class of systems than other conditions in the literature. The principal idea to obtain such generalized conditions is an appropriate selection of Lyapunov–Krasovskii functional that give us new stability conditions (less conservative) than other conditions in the literature. The delay-dependent robust stability and stabilization conditions will be presented in terms of LMI.

In this paper, we consider the problem of robust  $H\infty$  control design for T-S fuzzy model with time-delay. It is organized as follows; we introduce a class of T-S fuzzy model with time-delay in section 2. In section 3, state feedback fuzzy control law for fuzzy systems with time delay is proposed and based on the PDC and Lyapunov–Krasovskii functional using LMI. Section 4 establishes the delay-dependent stability and stabilization of uncertain T-S fuzzy models results in terms of LMI. One example of simulation in the Matlab environment is shown in section 5 to illustrate the effectiveness of our results. Finally, concluding remarks are given in section 6.

## 2 Problem statement

Consider a continuous-time uncertain fuzzy system with time state and input delays, which are represented by a following T-S fuzzy model [1].

*Plant Rule i*

IF  $z_1(t)$  is  $\mu_{i1}$  and ... and  $z_g(t)$  is  $\mu_{ig}$  THEN

$$\begin{cases} \dot{x}(t) = (A_i + \Delta A_i)x(t) + (A_{id} + \Delta A_{id})x(t - \tau_1(t)) \\ \quad + (B_i + \Delta B_i)u(t) + (B_{id} + \Delta B_{id})u(t - \tau_2(t)) + B_{iw}W(t) \\ y(t) = C_i x(t) + C_{id} x(t - \tau_1(t)) + D_i u(t) + B_{iz} W(t) \\ x(t) = \phi(t), \quad t \in [-\max(\tau_{01}, \tau_{02}), 0], \quad i = 1, 2, \dots, r \end{cases} \quad (1)$$

where  $r$  is the number of IF-THEN rules,  $\mu_{ij}$  ( $i = 1, 2, \dots, r$ ;  $j = 1, 2, \dots, g$ ) are fuzzy sets and  $z(t) = [z_1(t) \ z_2(t) \dots z_p(t)]^T \in R^p$  are known as premise variables depending linearly or not on

$x(t)$ ,  $A_i$ ,  $A_{id}$ ,  $B_i$ ,  $B_{id}$ ,  $C_i$ ,  $C_{id}$ ,  $B_{iw}$ ,  $B_{iz}$  and  $D_i$  are known constant matrices with appropriate dimensions,  $x(t) \in R^n$  is the state variable vector;  $u(t) \in R^m$  is the control input vector,  $y(t) \in R^q$  is the controlled output,  $W(t) \in R^p$  which is assumed to belong to  $L_2 \in [0, \infty]$ , denotes the external perturbation.  $\tau_1(t)$  and  $\tau_2(t)$  are the continuous functions represents a time-varying delays in state and input respectively, satisfying;

$$0 \leq \tau_i(t) \leq \tau_{0i} \leq \infty, \quad \dot{\tau}_i(t) \leq \mu_i \leq \infty, \quad (i = 1, 2) \quad \forall t \geq 0$$

where  $\tau_{0i}$  and  $\mu_i$  are constants.  $\Delta A_i$ ,  $\Delta A_{id}$ ,  $\Delta B_i$  and  $\Delta B_{id}$  are time-varying matrices with appropriate dimensions, defined as;

$$\begin{aligned} \Delta A_i &= H_i F_i(t) E_{ai}, \quad \Delta A_{id} = H_i F_i(t) E_{aid}, \quad \Delta B_i = H_i F_i(t) E_{bi} \\ \Delta B_{id} &= H_i F_i(t) E_{bid} \end{aligned} \quad (2)$$

where  $H_i$ ,  $E_{ai}$ ,  $E_{aid}$ ,  $E_{bi}$  and  $E_{bid}$  are known constant real matrices with appropriate dimensions that characterize the structures of uncertainties,  $F_i(t)$  is an unknown real matrix function satisfying the inequality:

$$F_i^T(t) F_i(t) \leq I \quad (3)$$

By using center-average defuzzifier, product inference and singleton fuzzifier, the final outputs of the T-S fuzzy systems (1) can be inferred as follows

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^r h_i(z(t)) \times \\ \quad ((A_i + \Delta A_i)x(t) + (A_{id} + \Delta A_{id})x(t - \tau_1(t)) \\ \quad + (B_i + \Delta B_i)u(t) + (B_{id} + \Delta B_{id})u(t - \tau_2(t)) + B_{iw}W(t)) \\ y(t) = \sum_{i=1}^r h_i(z(t)) \times \\ \quad C_i x(t) + C_{id} x(t - \tau_1(t)) + D_i u(t) + B_{iz} W(t) \end{cases} \quad (4)$$

where

$$h_i(z(t)) = w_i(z(t)) / \sum_{i=1}^r w_i(z(t)), \quad w_i(z(t)) = \prod_{j=1}^g \mu_{ij}(z(t))$$

The term  $\mu_{ij}(z(t))$  is the degree of membership of  $z_j(t)$  in fuzzy set  $\mu_{ij}$ . In this paper, it is assumed that  $w_i(z(t)) \geq 0$   $i = 1, \dots, r$  and  $\sum_{i=1}^r w_i(z(t)) > 0$  for all  $t$ . Therefore,

$$h_i(z(t)) \geq 0 \quad i = 1, \dots, r \quad \text{and} \quad \sum_{i=1}^r h_i(z(t)) = 1.$$

In this paper, a state feedback T-S fuzzy-model-based controller will be designed for stabilizing T-S fuzzy system (4) via PDC low.

This law of control shares the same fuzzy sets than the T-S fuzzy model. The gains of the controller can be determined by using a LMI formulation.

*Control rule i:*

IF  $z_1(t)$  is  $\mu_{i_1}$  and, ...,  $z_g(t)$  is  $\mu_{i_g}$  THEN

$$u(t) = K_i x(t) \quad i = 1, \dots, r \quad (5)$$

Hence, the overall fuzzy control law is represented by:

$$u(t) = \sum_{i=1}^r h_i(z(t)) K_i x(t) \quad (6)$$

where  $K_i (i = 1, 2, \dots, r)$  are the local control gains to be determined later.

*Remark 1:* The existence of input delay leads to the term  $u(t - \tau_2) = \sum_{i=1}^r h_i(z(t - \tau_2)) K_i x(t - \tau_2)$ , so, it is natural and necessary to make an assumption that the functions  $h_i(z(t))$   $i = 1, \dots, r$  are well defined for all  $t \in [-\tau_2, 0]$  and also satisfy the equality  $h_i(z(t - \tau_2)) \geq 0$   $i = 1, \dots, r$  and  $\sum_{i=1}^r h_i(z(t - \tau_2)) = 1$ . For notational simplicity, we use  $h_i$  and  $h_i^{r^2}$  to represent  $h_i(z(t))$  and  $h_i(z(t - \tau_2))$  respectively in the following description.

The design of the fuzzy controller is to determine the feedback gains  $K_i (i = 1, 2, \dots, r)$  such that the resulting closed-loop system is asymptotically stable.

Associated with the control law (6), the resulting closed-loop system can be expressed as follows:

$$\dot{x}(t) = \sum_{i=1}^r \sum_{j=1}^r h_i h_j^{r^2} \times (\bar{A}_i x(t) + \bar{A}_{id} x(t - \tau_1(t)) + \bar{B}_i K_j x(t) + \bar{B}_{id} K_j x(t - \tau_2(t)) + B_{iw} W(t))$$

Then

$$\left\{ \begin{array}{l} \dot{x}(t) = \sum_{i=1}^r h_i^2 [(\bar{A}_i + \bar{B}_i K_j) x(t) + \bar{A}_{id} x(t - \tau_1(t)) + B_{iw} W(t)] \\ \quad + \sum_{j=1}^r h_i h_j^{r^2} [\bar{B}_{id} K_j x(t - \tau_2(t))] \\ y(t) = \sum_{i=1}^r \sum_{j=1}^r h_i [(C_i + D_i K_j) x(t) + C_{id} x(t - \tau_1(t)) + B_{iz} W(t)] \end{array} \right. \quad (7)$$

$$\begin{aligned} \bar{A}_i &= A_i + \Delta A_i, \quad \bar{A}_{id} = A_{id} + \Delta A_{id}, \quad \bar{B}_i = B_i + \Delta B_i \quad \text{and} \\ \bar{B}_{id} &= B_{id} + \Delta B_{id}. \end{aligned}$$

For a prescribed scalar  $J > 0$ , the performance index  $J$  is defined as

$$J = \int_0^\infty (y^T(s)y(s) - \gamma^2 W^T(s)W(s))ds \quad (8)$$

The purpose of this work is to design a robust  $H_\infty$  controller (6) for the T-S fuzzy model (7) such that the following requirement is satisfied:

- Under the zero initial condition, system (7) with controller (6) satisfies  $\|y(t)\|_2 < \gamma \|W(t)\|_2$  for any nonzero  $W(t) \in L_2[0, \infty]$ , where  $\gamma > 0$  is a prescribed scalar.

### 3 Design of robust $H_\infty$ controller and stabilization analysis

In this section, we will derive the delay-dependent LMI conditions for  $H_\infty$  performance analysis for the systems (7).

The objective of this paper is to determine the feedback control law  $u(t) = \sum_{i=1}^r h_i(z(t)) K_i x(t)$  such that systems (7) to be robust asymptotically stable with an  $H_\infty$  norm bound  $\gamma > 0$ . Before stating our main results, the following lemmas are first presented, which will be used in the proofs of our results.

*Lemma 1:* For any constant matrix  $M > 0$ , any scalars  $a$  and  $b$  with  $a < b$ , and any vector function  $x(t) : [a, b] \rightarrow R^n$  such that the integrals concerned are well defined, then, the following inequality holds [11]:

$$\left[ \int_a^b x(s) ds \right]^T M \left[ \int_a^b x(s) ds \right] \leq (b - a) \int_a^b x^T(s) M x(s) ds \quad (9)$$

*Lemma 2:* For any constant matrices  $Q_{11}$ ,  $Q_{22}$ ,

$$Q_{12} \in R^{n \times n}, \quad Q_{11} \geq 0, \quad Q_{22} \geq 0, \quad \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} \geq 0, \quad \text{scalar } \tau(t) \leq \tau_0$$

and vector function  $\dot{x} : [-\tau_0, 0] \rightarrow R^n$  such that the following integration is well defined, then [13];

$$\begin{aligned} -\tau_0 \int_{t-\tau_0}^t \begin{bmatrix} x^T(s) & \dot{x}^T(s) \end{bmatrix} \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix} ds \leq \\ \begin{bmatrix} x(t) \\ x(t-\tau) \\ \vdots \\ \int_{t-\tau(t)}^t x(s) ds \end{bmatrix}^T \begin{bmatrix} -Q_{22} & Q_{22} & -Q_{12}^T \\ Q_{22} & -Q_{22} & Q_{12}^T \\ * & * & -Q_{11} \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\tau) \\ \vdots \\ \int_{t-\tau(t)}^t x(s) ds \end{bmatrix} \end{aligned} \quad (10)$$

*Lemma3:* Let  $Q = Q^T D, E$  and  $F(t)$  be real matrices of appropriate dimensions and  $F(t)$  satisfying  $F^T(t)F(t) \leq I$  then, the following inequality [14];

$$Q + DF(t)E + E^T F^T(t)D^T \leq 0 \quad (11)$$

is true, if and only if the following inequality holds for  $\lambda > 0$

$$Q + \lambda DD^T + \lambda^{-1} E^T E \leq 0 \quad (12)$$

Our first theorem is given as follows;

*Theorem1:* For given scalars  $\tau_{0i} > 0, \mu_i > 0, (i=1,2)$  and  $\nu_i > 0, (i=1-4)$  as well as the given matrices  $K_i (i=1, \dots, r)$  systems (7) is asymptotically stable via PDC technique for any time-delay  $\tau_i(t)$  satisfying  $0 \leq \tau_i(t) \leq \tau_{0i}$  ( $i=1,2$ ) if there exist matrices  $M_{11} > 0, M_{22} > 0, N_{11} > 0, N_{22} > 0, V_{11} > 0, V_{22} > 0, \tilde{W}_{11} > 0, W_{22} > 0, S_1, S_2$ , and real matrices  $T_i (i=1-4), M_{12}, N_{12}, V_{12}, W_{12}, K_j (j=1,2, \dots, r)$  such that the following conditions hold for  $i, j = 1, \dots, r$  ( $i \leq j$ ).

$$\Omega_{ii} < 0 \quad i = 1, 2, \dots, r \quad (13)$$

$$\Omega_{ij} + \Omega_{ji} < 0 \quad i < j < r \quad (14)$$

$$\begin{bmatrix} M_{11} & M_{12} \\ * & M_{22} \end{bmatrix} \geq 0, \begin{bmatrix} N_{11} & N_{12} \\ * & N_{22} \end{bmatrix} \geq 0, \begin{bmatrix} V_{11} & V_{12} \\ * & V_{22} \end{bmatrix} \geq 0, \begin{bmatrix} W_{11} & W_{12} \\ * & W_{22} \end{bmatrix} \geq 0 \quad (15)$$

with

$$\Omega_y = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & \Omega_{15} & \Omega_{16} & \Omega_{17} \\ * & \Omega_{22} & \Omega_{23} & \Omega_{24} & \Omega_{25} & \Omega_{26} & \Omega_{27} \\ * & * & \Omega_{33} & \Omega_{34} & \Omega_{35} & \Omega_{36} & \Omega_{37} \\ * & * & * & \Omega_{44} & \Omega_{45} & \Omega_{46} & \Omega_{47} \\ * & * & * & * & \Omega_{55} & \Omega_{56} & \Omega_{57} \\ * & * & * & * & * & \Omega_{66} & \Omega_{67} \\ * & * & * & * & * & * & \Omega_{77} \end{bmatrix}$$

$$\begin{aligned} \Omega_{11} &= M_{12} + M_{12}^T + N_{12} + N_{12}^T - V_{22} - W_{22} + S_1 + S_2 + \tau_{01}^2 V_{11} + \tau_{02}^2 W_{11} \\ &\quad + T_1 (\bar{A}_i + \bar{B}_i K_j) + (\bar{A}_i + \bar{B}_i K_j)^T T_1^T + (C_i + D_i K_j)^T (C_i + D_i K_j) \\ \Omega_{12} &= -c_1 M_{12} + V_{22} + T_1 \bar{A}_{id} + (\bar{A}_i + \bar{B}_i K_j)^T T_2^T + (C_i + D_i K_j)^T C_{id} \\ \Omega_{13} &= M_{22}^T - V_{12}^T, \\ \Omega_{14} &= -c_2 N_{12} + W_{22} + T_1 \bar{B}_{id} K_j + (\bar{A}_i + \bar{B}_i K_j)^T T_3^T \\ \Omega_{15} &= N_{22}^T - W_{12}^T \\ \Omega_{16} &= M_{11} + N_{11} + \tau_{01}^2 V_{12} + \tau_{02}^2 W_{12} + (\bar{A}_i + \bar{B}_i K_j)^T T_4^T - T_1 \\ \Omega_{17} &= T_1 B_{iw} + (C_i + D_i K_j)^T B_{iz} \\ \Omega_{22} &= -c_1 S_1 - V_{22} + T_2 \bar{A}_{id} + \bar{A}_{id}^T T_2^T + C_{id}^T C_{id} \end{aligned}$$

$$\begin{aligned} \Omega_{23} &= -c_1 M_{22}^T - V_{12}^T, \Omega_{24} = T_2 \bar{B}_{id} K_j + \bar{A}_{id}^T T_3^T, \Omega_{25} = 0 \\ \Omega_{26} &= -T_2 + \bar{A}_{id}^T T_4^T, \quad \Omega_{27} = T_2 B_{iw} + C_{id}^T B_{iz}, \quad \Omega_{33} = -V_{11}, \\ \Omega_{34} &= 0, \Omega_{35} = 0, \Omega_{36} = M_{12}^T, \Omega_{37} = 0, \\ \Omega_{44} &= -c_2 S_2 - W_{22} + T_3 \bar{B}_{id} K_j + K_j^T \bar{B}_{id}^T T_3^T, \Omega_{45} = -c_2 N_{22}^T + W_{12}^T, \\ \Omega_{46} &= -T_3 + K_j^T B_{id}^T T_4^T, \Omega_{47} = T_3 B_{iw} \\ \Omega_{ij55} &= -W_{11}, \Omega_{ij56} = 0, \Omega_{ij57} = 0, \\ \Omega_{66} &= \tau_{01}^2 V_{22} + \tau_{02}^2 W_{22} - T_4 - T_4^T \\ \Omega_{67} &= T_4 B_{iw}, \quad \Omega_{77} = B_{iz}^T B_{iz} - \gamma^2 I \end{aligned}$$

*Proof:* To prove the theorem, we consider the following Lyapunov–Krasovskii functional candidate:

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) + V_5(t) + V_6(t) \quad (16)$$

where

$$\begin{aligned} V_1(t) &= \mathcal{G}_1^T(t) M \mathcal{G}(t) \\ V_2(t) &= \mathcal{G}_2^T(t) N \mathcal{G}(t) \\ V_3(t) &= \int_{t-\tau_1(t)}^t x^T(s) S_1 x(s) ds \\ V_4(t) &= \int_{t-\tau_2(t)}^t x^T(s) S_2 x(s) ds \\ V_5(t) &= \tau_{01} \int_{t-\tau_{01}(t)}^t (s-t+\tau_{01}) \phi^T(s) V \phi(s) ds \\ V_6(t) &= \tau_{02} \int_{t-\tau_{02}(t)}^t (s-t+\tau_{02}) \phi^T(s) W \phi(s) ds \end{aligned}$$

with

$$\begin{aligned} \mathcal{G}_1(t) &= \left[ x^T(t) \quad \left( \int_{t-\tau_1(t)}^t x(s) ds \right)^T \right]^T, M = \begin{bmatrix} M_{11} & M_{12} \\ * & M_{22} \end{bmatrix} \\ \mathcal{G}_2(t) &= \left[ x^T(t) \quad \left( \int_{t-\tau_2(t)}^t x(s) ds \right)^T \right]^T, N = \begin{bmatrix} N_{11} & N_{12} \\ * & N_{22} \end{bmatrix} \\ \phi(t) &= \left[ x^T(s) \quad \dot{x}^T(s) \right]^T, V = \begin{bmatrix} V_{11} & V_{12} \\ * & V_{22} \end{bmatrix}, W = \begin{bmatrix} W_{11} & W_{12} \\ * & W_{22} \end{bmatrix} \end{aligned}$$

and

$$\begin{aligned} M &= M^T > 0, \quad N = N^T > 0, \quad V = V^T \geq 0, \quad W = W^T \geq 0; \\ S_1 &\geq 0, \quad S_2 \geq 0, \quad c_1 = (1-\mu_1), \quad c_2 = (1-\mu_2) \end{aligned}$$

The time derivative of  $V(t)$  is taken along state trajectory (7), yielding;

$$\begin{aligned} \dot{V}_1(t) &= \dot{\mathcal{G}}_1^T(t) M \mathcal{G}(t) + \mathcal{G}_1^T(t) M \dot{\mathcal{G}}(t) = 2 \mathcal{G}_1^T(t) M \dot{\mathcal{G}}(t) \leq \\ &\leq 2 \left[ x^T(t) \quad \left( \int_{t-\tau_1(t)}^t x(s) ds \right)^T \right] \begin{bmatrix} M_{11} & M_{12} \\ * & M_{22} \end{bmatrix} \frac{d}{dt} \left[ \begin{array}{c} x(t) \\ \left( \int_{t-\tau_1(t)}^t x(s) ds \right) \end{array} \right] \leq \\ &\leq 2 \left[ x^T(t) \quad \left( \int_{t-\tau_1(t)}^t x(s) ds \right)^T \right] \begin{bmatrix} M_{11} & M_{12} \\ * & M_{22} \end{bmatrix} \left[ \begin{array}{c} \dot{x}(t) \\ x(t) - c_1 x(t - \tau_1) \end{array} \right] \\ &\leq x^T(t) M_{11} \dot{x}(t) + \dot{x}^T(t) M_{11} x(t) + \left( \int_{t-\tau_1(t)}^t x(s) ds \right)^T M_{12}^T \dot{x}(t) + \\ &\quad \dot{x}^T(t) M_{12} \left( \int_{t-\tau_1(t)}^t x(s) ds \right) + x^T(t) M_{12} x(t) + x^T(t) M_{12}^T x(t) \end{aligned}$$

$$\begin{aligned}
 & + \left( \int_{t-\tau_1(t)}^t x(s) ds \right)^T M_{22} x(t) + x^T(t) M_{22} \left( \int_{t-\tau_1(t)}^t x(s) ds \right) \\
 & - c_1 x^T(t) M_{12} x(t - \tau_1) - c_1 x^T(t - \tau_1) M_{12}^T x(t) \\
 & - c_1 \left( \int_{t-\tau_1(t)}^t x(s) ds \right)^T M_{22} x(t - \tau_1) \\
 & - c_1 x^T(t - \tau_1) M_{22} \left( \int_{t-\tau_1(t)}^t x(s) ds \right)
 \end{aligned}$$

From same method we obtains

$$\begin{aligned}
 \dot{V}_2(t) &= \dot{\vartheta}_2^T(t) N \vartheta(t) + \vartheta_2^T(t) N \dot{\vartheta}(t) = 2 \vartheta_2^T(t) N \dot{\vartheta}(t) \leq \\
 & 2 \left[ x^T(t) \left( \int_{t-\tau_2(t)}^t x(s) ds \right)^T \begin{bmatrix} N_{11} & N_{12} \\ * & N_{22} \end{bmatrix} \begin{bmatrix} \dot{x}(t) \\ x(t) - c_2 x(t - \tau_2) \end{bmatrix} \right] \\
 & \leq x^T(t) N_{11} \dot{x}(t) + \dot{x}^T(t) N_{11} x(t) + \left( \int_{t-\tau_2(t)}^t x(s) ds \right)^T N_{12}^T \dot{x}(t) \\
 & + \dot{x}^T(t) N_{12} \left( \int_{t-\tau_2(t)}^t x(s) ds \right) + x^T(t) N_{12} x(t) + x^T(t) N_{12}^T x(t) \\
 & + \left( \int_{t-\tau_2(t)}^t x(s) ds \right)^T N_{22} x(t) + x^T(t) N_{22} \left( \int_{t-\tau_2(t)}^t x(s) ds \right) \\
 & - c_2 x^T(t) N_{12} x(t - \tau_2) - c_2 x^T(t - \tau_2) N_{12}^T x(t) \\
 & - c_2 \left( \int_{t-\tau_2(t)}^t x(s) ds \right)^T N_{22} x(t - \tau_2) \\
 & - c_2 x^T(t - \tau_2) N_{22} \left( \int_{t-\tau_2(t)}^t x(s) ds \right) \\
 \dot{V}_3(t) &= x^T(t) S_1 x(t) - (1 - \dot{\tau}_1) x^T(t - \tau_1) S_1 x(t - \tau_1) \\
 & \leq x^T(t) S_1 x(t) - (1 - \mu_1) x^T(t - \tau_1) S_1 x(t - \tau_1) \\
 \dot{V}_4(t) &= x^T(t) S_2 x(t) - (1 - \dot{\tau}_2) x^T(t - \tau_2) S_2 x(t - \tau_2) \\
 & \leq x^T(t) S_2 x(t) - (1 - \mu_2) x^T(t - \tau_2) S_2 x(t - \tau_1) \\
 \dot{V}_5(t) &= \tau_{01} \int_{t-\tau_{01}(t)}^t (s - t + \tau_{01}) \phi^T(s) V \phi(s) ds \\
 & \leq \tau_{01}^2 \phi^T(s) V \phi(s) - \tau_{01} \int_{t-\tau_{01}(t)}^t \phi^T(s) V \phi(s) ds
 \end{aligned}$$

By using lemma 2, we obtain;

$$\begin{aligned}
 & -\tau_{01} \int_{t-\tau_{01}(t)}^t \phi^T(s) V \phi(s) ds = \\
 & -\tau_{01} \int_{t-\tau_{01}(t)}^t \left[ x^T(s) \quad \dot{x}^T(s) \begin{bmatrix} V_{11} & V_{12} \\ V_{12}^T & V_{22} \end{bmatrix} \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix} \right] ds \\
 & \leq \left[ \begin{array}{c|c} x(t) & \left[ \begin{array}{ccc} -V_{22} & V_{22} & -V_{12}^T \\ V_{22} & -V_{22} & V_{12}^T \\ -V_{12} & V_{12} & -V_{11} \end{array} \right] \\ \hline x(t - \tau_1) & \left[ \begin{array}{c} x(t - \tau_1) \\ \vdots \\ \int_{t-\tau_1(t)}^t x(t) ds \end{array} \right] \end{array} \right] \left[ \begin{array}{c} x(t) \\ x(t - \tau_1) \\ \vdots \\ \int_{t-\tau_1(t)}^t x(t) ds \end{array} \right] \\
 \dot{V}_5(t) &= x^T(s) (\tau_{01}^2 V_{11}) x(s) + \dot{x}^T(s) (\tau_{01}^2 V_{12}^T) x(s) + \\
 & x^T(s) (\tau_{01}^2 V_{12}) \dot{x}(s) + \dot{x}^T(s) (\tau_{01}^2 V_{22}^T) \dot{x}(s) + x^T(t) (-V_{22}) x(t) + \\
 & x^T(t - \tau_1) (V_{22}) x(t) + \left( \int_{t-\tau_1(t)}^t x(t) ds \right)^T (-V_{12}) x(t) + \\
 & x^T(t) (V_{22}) x(t - \tau_1) + x^T(t - \tau_1) (-V_{22}) x(t - \tau_1) + \\
 & \left( \int_{t-\tau_1(t)}^t x(t) ds \right)^T (V_{12}) x(t - \tau_1) + x^T(t) (-V_{12}^T) \int_{t-\tau_1(t)}^t x(t) ds + \\
 & x^T(t - \tau_1) (V_{12}^T) \int_{t-\tau_1(t)}^t x(t) ds + \\
 & \left( \int_{t-\tau_1(t)}^t x(t) ds \right)^T (-V_{11}) \int_{t-\tau_1(t)}^t x(t) ds
 \end{aligned}$$

Using same method to calculate  $\dot{V}_6(t)$

$$\begin{aligned}
 \dot{V}_6(t) &= \tau_{02} \left[ \int_{t-\tau_{02}(t)}^t -\phi^T(s) W \phi(s) + (s - t + \tau_{02}) \frac{d}{dt} \phi^T(s) V \phi(s) \right] ds \\
 & \leq x^T(s) (\tau_{02}^2 W_{11}) x(s) + \dot{x}^T(s) (\tau_{02}^2 W_{12}^T) x(s) + x^T(s) (\tau_{02}^2 W_{12}) \dot{x}(s) \\
 & + \dot{x}^T(s) (\tau_{02}^2 W_{22}^T) \dot{x}(s) + x^T(t) (-W_{22}) x(t) + x^T(t - \tau_2) (W_{22}) x(t) \\
 & + \left( \int_{t-\tau_2(t)}^t x(t) ds \right)^T (-W_{12}) x(t) + x^T(t - \tau_2) (-W_{22}) x(t - \tau_2) \\
 & + x^T(t) (W_{22}) x(t - \tau_2) + \left( \int_{t-\tau_2(t)}^t x(t) ds \right)^T (W_{12}) x(t - \tau_2) \\
 & + \left( \int_{t-\tau_2(t)}^t x(t) ds \right)^T (-W_{11}) \int_{t-\tau_2(t)}^t x(t) ds \\
 & + \left( \int_{t-\tau_2(t)}^t x(t) ds \right)^T (-W_{12}) \int_{t-\tau_2(t)}^t x(t) ds
 \end{aligned}$$

Now define;

$$\zeta = \begin{bmatrix} x^T(t) & x^T(t - \tau_1) & \left( \int_{t-\tau_1}^t x(s) ds \right)^T & x^T(t - \tau_2) \\ \left( \int_{t-\tau_2}^t x(s) ds \right)^T & \dot{x}^T(t) & W^T(t) \end{bmatrix}^T \quad (17)$$

From  $\dot{V}_1(t), \dot{V}_2(t), \dot{V}_3(t), \dot{V}_4(t), \dot{V}_5(t), \dot{V}_6(t)$  we obtain;

$$\dot{V}(t) \leq \zeta^T \underbrace{\begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} & \Phi_{15} & \Phi_{16} & 0 \\ * & \Phi_{22} & \Phi_{23} & 0 & 0 & 0 & 0 \\ * & * & -V_{11} & 0 & 0 & M_{12}^T & 0 \\ * & * & * & \Phi_{44} & \Phi_{45} & 0 & 0 \\ * & * & * & * & -W_{11} & 0 & 0 \\ * & * & * & * & * & \Phi_{66} & 0 \\ * & * & * & * & * & * & 0 \end{bmatrix}}_{\Phi^{ij}} \zeta \quad (18)$$

$$\Phi_{11} = M_{12} + M_{12}^T + N_{12} + N_{12}^T - V_{22} - W_{22}^T + S_1 + S_2 + \tau_{01}^2 V_{11} + \tau_{02}^2 W_{11}$$

$$\Phi_{12} = -c_1 M_{12} + V_{22}$$

$$\Phi_{13} = M_{22}^T - V_{12}^T$$

$$\Phi_{14} = -c_2 N_{12} + W_{22}$$

$$\Phi_{15} = N_{22}^T - W_{12}^T$$

$$\Phi_{16} = M_{11} + N_{11} + \tau_{01}^2 V_{12} + \tau_{02}^2 W_{12}$$

$$\Phi_{22} = -c_1 S_1 - V_{22}$$

$$\Phi_{23} = -c_1 M_{22}^T - V_{12}^T$$

$$\Phi_{44} = -c_2 S_2 - W_{22}$$

$$\Phi_{45} = -c_2 N_{22}^T + W_{12}^T$$

$$\Phi_{66} = \tau_{01}^2 V_{22} + \tau_{02}^2 W_{22}$$

Next, we will introduce some free variables as following:

$$\begin{aligned}
 & 2 \times \left\{ x^T(t) T_1 + x^T(t - \tau_1) T_2 + x^T(t - \tau_2) T_3 + \dot{x}^T(t) T_4 \right\} \\
 & \left\{ \sum_{i=1}^r \sum_{j=1}^r h_i h_j^r \left[ \begin{array}{c} [(\bar{A}_i + \bar{B}_i K_j) x(t) + \bar{A}_{id} x(t - \tau_1(t))] + \\ \bar{B}_{id} K_j x(t - \tau_2(t)) + B_{iw} W(t) \end{array} \right] - \dot{x}(t) \right\} = 0
 \end{aligned}$$

This equality can be writing as follows

$$\sum_{i=1}^r \sum_{j=1}^r h_i h_j^r \zeta^T \left( \Theta_{ij} + \Lambda_{Hi} F_i \Lambda_{Eij} + \Lambda_{Eij}^T F_i^T \Lambda_{Hi}^T \right) \zeta = 0 \quad (19)$$

where

$$\begin{aligned} \Lambda_{Hi}^T &= \begin{bmatrix} H_i^T T_1^T & H_i^T T_2^T & 0 & H_i^T T_3^T & 0 & H_i^T T_4^T & 0 \end{bmatrix} \\ \Lambda_{Eij}^T &= \begin{bmatrix} E_{ai} + E_{bi} K_j & E_{aid} & 0 & E_{bid} K_j & 0 & 0 & 0 \end{bmatrix} \\ \Theta_{ij} &= \begin{bmatrix} T_1(A_i + B_i K_j) + & \left( \begin{array}{c} T_1 A_{id} + \\ (A_i + B_i K_j)^T T_1^T \end{array} \right) \\ * & T_2 A_{id} + A_{id}^T T_2^T \\ * & * \\ * & * \\ * & * \\ * & * \\ * & * \end{bmatrix} \quad (20) \\ 0 & \begin{pmatrix} T_1 B_{id} K_j + \\ (A_i + B_i K_j)^T T_3^T \end{pmatrix} \quad 0 \quad \begin{pmatrix} (A_i + B_i K_j)^T T_4^T \\ -T_1 \end{pmatrix} \quad T_1 B_{iw} \\ 0 & T_2 B_{id} K_j + A_{id}^T T_3^T \quad 0 \quad -T_2 + A_{id}^T T_4^T \quad T_2 B_{iw} \\ 0 & 0 \quad 0 \quad 0 \quad 0 \\ * & T_3 B_{id} K_j + K_j^T B_{id}^T T_3^T \quad 0 \quad -T_3 + K_j^T B_{id}^T T_4^T \quad T_3 B_{iw} \\ * & * \quad 0 \quad 0 \quad 0 \\ * & * \quad * \quad -T_4 - T_4^T \quad T_4 B_{iw} \\ * & * \quad * \quad * \quad 0 \end{pmatrix} = 0 \end{aligned}$$

According to the definition of  $y(t)$ , we have

$$\begin{aligned} \dot{V}(t) &\leq \sum_{i=1}^6 V_i(t) + \sum_{i=1}^r \sum_{j=1}^r h_i h_j^{r2} \zeta^T (\Theta_{ij} + \Lambda_{Hi} F_i \Lambda_{Eij} + \Lambda_{Eij}^T F_i^T \Lambda_{Hi}^T) \zeta \\ &\quad + y^T(t) y(t) - \gamma^2 W^T(t) W(t) \leq 0 \end{aligned}$$

By Lemma 3, it is obtained

$$\begin{aligned} \dot{V}(t) &\leq \sum_{i=1}^6 V_i(t) + \sum_{i=1}^r \sum_{j=1}^r h_i h_j^{r2} \zeta^T (\Theta_{ij} + \varepsilon_{ij} \Lambda_{Hi} \Lambda_{Hi}^T + \varepsilon_{ij}^{-1} \Lambda_{Eij}^T \Lambda_{Eij}) \zeta \\ &\quad + y^T(t) y(t) - \gamma^2 W^T(t) W(t) \leq 0 \end{aligned}$$

where

$$\begin{aligned} y^T(t) y(t) &= \quad (21) \\ 2 \sum_{i=1}^r \sum_{j<i}^r h_i h_j^{r2} [(C_i + D_i K_j) x(t) + C_{id} x(t - \tau_1(t)) + B_{ie} W(t)] \\ &\leq \sum_{i=1}^r \sum_{j<i}^r h_i h_j^{r2} \zeta^T \times \\ &\quad \begin{bmatrix} ((C_i + D_i K_j)^T) & ((C_i + D_i K_j)^T) & 0 & 0 & 0 & 0 & \left( \begin{array}{c} (C_i + D_i K_j)^T \\ \times (C_i + D_i K_j) \end{array} \right) \\ \times C_{id} & C_{id}^T C_{id} & 0 & 0 & 0 & 0 & C_{id}^T B_{ie} \\ * & * & 0 & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 & 0 \\ * & * & * & * & * & 0 & 0 \\ * & * & * & * & * & * & B_{ie}^T B_{ie} \end{bmatrix} \zeta \\ &\leq \sum_{i=1}^r \sum_{j<i}^r h_i h_j^{r2} \zeta^T \times G_{ij}^T G_{ij} \zeta \quad (22) \end{aligned}$$

where  $G_{Hi}^T = [(C_i + D_i K_j)^T \quad C_{id}^T \quad 0 \quad 0 \quad 0 \quad 0 \quad B_{ie}^T]^T$   
From (18)-(21) and (22) we can obtain  $\dot{V}(t)$  as;

$$\dot{V}(t) \leq \sum_{i=1}^r \sum_{j=1}^r h_i h_j^{r2} \zeta^T \Omega_{ij} \zeta \quad (23)$$

From (23), we arrive at (13)- (14) and (15). This completes the proof.

**Theorem2:** For given scalars  $\tau_{0i} > 0$ ,  $\mu_i > 0$ , ( $i=1,2$ ) and  $\nu_i > 0$ , ( $i=1-4$ ) as well as the given matrices  $K_i$  ( $i=1, \dots, r$ ) systems (4) is asymptotically stable via PDC technique for any time-delay  $\tau_i(t)$  satisfying  $0 \leq \tau_i(t) \leq \tau_{0i}$  ( $i=1,2$ ) if there exist matrices  $\bar{M}_{11} > 0$ ,  $\bar{M}_{22} > 0$ ,  $\bar{N}_{11} > 0$ ,  $\bar{N}_{22} > 0$ ,  $\bar{V}_{11} > 0$ ,  $\bar{V}_{22} > 0$ ,  $\bar{W}_{11} > 0$ ,  $\bar{W}_{22} > 0$ ,  $\bar{S}_1$ ,  $\bar{S}_2$  and real matrices  $X$ ,  $\bar{M}_{12}$ ,  $\bar{N}_{12}$ ,  $\bar{V}_{12}$ ,  $\bar{W}_{12}$ ,  $K_j$  ( $j=1,2,\dots,r$ ) such that the following LMIs hold for  $i, j = 1, \dots, r$  ( $i \leq j$ )

$$\begin{bmatrix} \Omega_{ii} & G_{ii}^T & \Lambda_{Hi} & \Lambda_{Eii}^T \\ * & -I & 0 & 0 \\ * & * & -\varepsilon_{ii} I & 0 \\ * & * & * & -\varepsilon_{ii} I \end{bmatrix} < 0 \quad i = 1, 2, \dots, r \quad (24)$$

$$\begin{bmatrix} \Omega_{ij} & G_{ij}^T & \Lambda_{Hj} & \Lambda_{Eij}^T \\ * & -I & 0 & 0 \\ * & * & -\varepsilon_{ij} I & 0 \\ * & * & * & -\varepsilon_{ij} I \end{bmatrix} + \begin{bmatrix} \Omega_{ji} & G_{ji}^T & \Lambda_{Hj} & \Lambda_{Eji}^T \\ * & -I & 0 & 0 \\ * & * & -\varepsilon_{ji} I & 0 \\ * & * & * & -\varepsilon_{ji} I \end{bmatrix} < 0 \quad (25)$$

$$\begin{bmatrix} \bar{M}_{11} & \bar{M}_{12} \\ * & \bar{M}_{22} \end{bmatrix} \geq 0, \begin{bmatrix} \bar{N}_{11} & \bar{N}_{12} \\ * & \bar{N}_{22} \end{bmatrix} \geq 0, \begin{bmatrix} \bar{V}_{11} & \bar{V}_{12} \\ * & \bar{V}_{22} \end{bmatrix} \geq 0, \begin{bmatrix} \bar{W}_{11} & \bar{W}_{12} \\ * & \bar{W}_{22} \end{bmatrix} \geq 0 \quad (26)$$

where

$$\begin{aligned} \Omega_{11} &= \bar{M}_{12} + \bar{M}_{12}^T + \bar{N}_{12} + \bar{N}_{12}^T - \bar{V}_{22} - \bar{W}_{22} + \bar{S}_1 + \bar{S}_2 + \tau_{01}^2 \bar{V}_{11} + \\ &\quad \tau_{02}^2 \bar{W}_{11} + A_i X^T + B_i \bar{K}_j) + X A_i^T + \bar{K}_j^T B_i^T \\ \Omega_{12} &= -c_1 \bar{M}_{12} + \bar{V}_{22} + \bar{A}_{id} X^T + \nu_1 X A_i^T + \nu_2 \bar{K}_j^T B_i^T \\ \Omega_{13} &= \bar{M}_{22}^T - \bar{V}_{12}^T, \\ \Omega_{14} &= -c_2 \bar{N}_{12} + W_{22} + B_{id} \bar{K}_j + \nu_3 X A_i^T + \nu_3 \bar{K}_j^T B_i^T \\ \Omega_{15} &= \bar{N}_{22}^T - \bar{W}_{12}^T \\ \Omega_{16} &= \bar{M}_{11} + \bar{N}_{11} + \tau_{01}^2 \bar{V}_{12} + \tau_{02}^2 \bar{W}_{12} + \nu_4 X A_i^T + \nu_4 \bar{K}_j^T B_i^T - X^T \\ \Omega_{17} &= B_{iw}, \quad \Omega_{ij22} = -c_1 \bar{S}_1 - \bar{V}_{22} + \nu_2 A_{id} X^T + \nu_2 X A_{id}^T \\ \Omega_{23} &= -c_1 \bar{M}_{22}^T - \bar{V}_{12}^T, \quad \Omega_{ij24} = \nu_2 B_{id} \bar{K}_j + \nu_2 X \bar{A}_{id}^T, \quad \Omega_{25} = 0 \\ \Omega_{26} &= -\nu_2 X^T + \nu_4 X A_{id}^T \\ \Omega_{27} &= \nu_{22} B_{iw}, \quad \Omega_{33} = -\bar{V}_{11}, \quad \Omega_{34} = 0, \quad \Omega_{35} = 0 \\ \Omega_{36} &= \bar{M}_{12}^T, \quad \Omega_{37} = 0 \\ \Omega_{ij44} &= -c_2 \bar{S}_2 - \bar{W}_{22} + \nu_3 \bar{B}_{id} \bar{K}_j + \nu_3 \bar{K}_j^T \bar{B}_{id}^T \\ \Omega_{45} &= -c_2 \bar{N}_{22}^T + \bar{W}_{12}^T, \quad \Omega_{46} = -\nu_3 X^T + \nu_4 \bar{K}_j^T B_{id}^T, \quad \Omega_{47} = \nu_3 B_{iw} \\ \Omega_{55} &= -\bar{W}_{11}, \quad \Omega_{56} = 0, \quad \Omega_{57} = 0, \\ \Omega_{66} &= \tau_{01}^2 \bar{V}_{22} + \tau_{02}^2 \bar{W}_{22} - \nu_4 X^T T_4 - \nu_4 X \\ \Omega_{67} &= \nu_4 B_{iw}, \quad \Omega_{77} = -\gamma^2 I \end{aligned}$$

**Proof:** Noting that (13) and (14) are not an LMI, we cannot solve them directly using

Matlab LMI Toolbox. In order to solve (13) and (14) efficiently, pre- and post-multiply both sides of (13) and (14) with  $\text{diag}[X \ X \ X \ X \ X \ X \ I \ I \ I \ I]$  and their transpose, respectively. And define new variables

$$\begin{aligned} X &= T_1^{-1}, \bar{M}_{11} = XM_{11}X^T, \bar{M}_{12} = XM_{12}X^T, \bar{M}_{22} = XM_{22}X^T \\ \bar{N}_{11} &= XN_{11}X^T, \bar{N}_{12} = XN_{12}X^T, \bar{N}_{22} = XN_{22}X^T \\ \bar{V}_{11} &= XV_{11}X^T, \bar{V}_{12} = XV_{12}X^T, \bar{V}_{22} = XV_{22}X^T \\ \bar{W}_{11} &= XW_{11}X^T, \bar{W}_{12} = XW_{12}X^T, \bar{W}_{22} = XW_{22}X^T \\ \bar{S}_1 &= XS_1X^T, \bar{S}_2 = XS_2X^T, \bar{K}_j = K_jX^T \quad (j=1,\dots,r) \end{aligned}$$

$$T_2 = v_2 T_1, T_3 = v_3 T_1, T_4 = v_2 T_1$$

we can obtain (24)-(25) and (26), respectively. This completes the proof,

Moreover, the control gain  $K_j$  is given by

$$K_j = \bar{K}_j(X^T)^{-1}, \quad j=1,2,\dots,r \quad (27)$$

## 4 Illustrative Example

In this section, we provide a numerical example and simulation to illustrate the effectiveness of the methods presented in this paper. We employed the stability conditions in Theorem 2 to obtain the feedback gains of the fuzzy controller to stabilize a time-delayed uncertain nonlinear system. Consider the following truck-trailer model with a time-state and input delay represented by T-S fuzzy model [10]:

*Rule1:*

$$\text{IF } z(t) = x_2 + a \frac{v\bar{t}}{Lt_0} x_1(t) + (1-a) \frac{v\bar{t}}{2L} x_1(t-\tau_1) \text{ is } h_1$$

*THEN*

$$\dot{x}(t) = A_1 x(t) + A_{1d} x(t-\tau_1(t)) + B_{1d} u(t-\tau_2(t))$$

*Rule2:*

$$\text{IF } z(t) = x_2 + a \frac{v\bar{t}}{Lt_0} x_1(t) + (1-a) \frac{v\bar{t}}{2L} x_1(t-\tau_1) \text{ is } h_2$$

*THEN*

$$\dot{x}(t) = A_2 x(t) + A_{2d} x(t-\tau_1(t)) + B_{2d} u(t-\tau_2(t))$$

here  $x(t) = [x_1(t) \ x_2(t) \ x_3(t)]^T$  and

$$\begin{aligned} B_1 &= B_2 = [v\bar{t}/lt_0 \ 0 \ 0]^T \\ B_{1w} &= B_{2w} = [0.02 \ 0; \ 0 \ 0.02; \ 0 \ 0.02] \\ B_{1d} &= B_{2d} = 0.1 \times B_1, B_{1z} = B_{2z} = [0 \ 1] \end{aligned}$$

$$\begin{aligned} A_1 &= \begin{bmatrix} -a \frac{v\bar{t}}{Lt_0} & 0 & 0 \\ a \frac{v\bar{t}}{Lt_0} & 0 & 0 \\ a \frac{v^2\bar{t}^2}{2Lt_0} & \frac{v\bar{t}}{t_0} & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -a \frac{v\bar{t}}{Lt_0} & 0 & 0 \\ a \frac{v\bar{t}}{Lt_0} & 0 & 0 \\ ad \frac{v^2\bar{t}^2}{2Lt_0} & \frac{dv\bar{t}}{t_0} & 0 \end{bmatrix} \\ A_{1d} &= \begin{bmatrix} -(1-a) \frac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-a) \frac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-a) \frac{v^2\bar{t}^2}{2Lt_0} & 0 & 0 \end{bmatrix}, \quad A_{2d} = \begin{bmatrix} -(1-a) \frac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-a) \frac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-a)d \frac{v^2\bar{t}^2}{2Lt_0} & 0 & 0 \end{bmatrix} \\ E_{a1d} &= E_{a2d} = [1 \ 0 \ 0] \\ H_1 &= H_2 = [0.255 \ 0.255 \ 0.255]^T, E_{b1} = E_{b2} = 0.3 \\ E_{b1d} &= E_{b2d} = 1 \\ E_{a1} &= E_{a2} = [0.03 \ 0.03 \ 0.03], F(t) = 2 \times \sin(t) \end{aligned}$$

and

$$a = 0.7, l = 2.8, L = 5.5, v = -1, t_0 = 0.5, \bar{t} = 2$$

$$d = 10t_0/\pi$$

The disturbance input is assumed to be

$$W(t) = \sin(2t) \times \exp(-0.2t)$$

The membership functions  $h_1$  and  $h_2$  are chosen as:

$$h_1 = \left(1 - \frac{1}{1 + \exp(-2z - 0.5\pi)}\right) \left(\frac{1}{1 + \exp(-2z + 0.5\pi)}\right),$$

$$h_2 = 1 - h_1$$

in the simulations, we use  $\phi(t) = [1 \ -1 \ 1]^T$ , let  $a = 0.5$ ,  $v_2 = 0.5$ ,  $v_3 = 0.2$  and  $v_4 = 1.2$ .

By applying the stability conditions in Theorem 2, with the aid of the Matlab LMI control toolbox, the design problem to determine the feedback gains can be defined as follows:

*Find*

$\bar{M}_{11} > 0, \bar{M}_{22} > 0, \bar{N}_{11} > 0, \bar{N}_{22} > 0, \bar{V}_{11} > 0, \bar{V}_{22} > 0, \bar{W}_{11} > 0$   
 $\bar{W}_{22} > 0, \bar{S}_1, \bar{S}_2$  and real matrices  $X, \bar{M}_{12}, \bar{N}_{12}, \bar{V}_{12}, \bar{W}_{12}$  and  $\bar{K}_{1,2}$

*Satisfying  $P > 0$ , (24), (25) and (26)*

We obtains the following fuzzy control law with  $\tau_{01} = 0.1$ ,  $\tau_{02} = 0.2$ ;

$$\begin{aligned} u(t) &= h_1 [2.6748 \ -3.8269 \ 0.8130] x(t) \\ &\quad + h_2 [3.1411 \ -5.2073 \ 1.0026] x(t) \end{aligned}$$

The response of the closed-loop system (7) with control law (6) is shown in Fig.1 under initial condition  $\phi(t) = [1 \ -1 \ 1]^T$ . The response of controller is shown in Fig.2. It shows that the closed-loop fuzzy system with time- delays has been effectively controlled using the designed fuzzy controller and the state response of this system has been uniformly asymptotically stable.

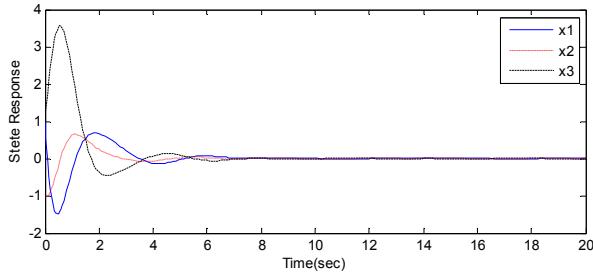


Fig.1. Response of the closed-loop system in Example 1 under fuzzy controller (6)

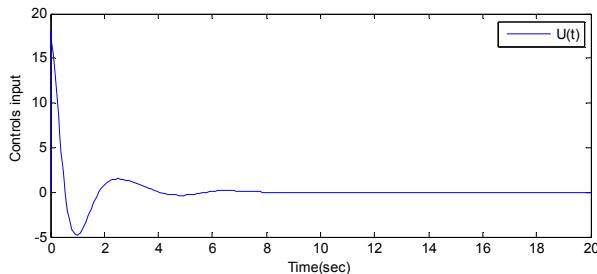


Fig.2. Controls input  $u(t)$

## 5 Conclusion

In this paper, we have discussed the robust stabilization analysis and  $H_\infty$  controller design problems for uncertain nonlinear systems described by T-S fuzzy models with time-state and input delay. Based on Lyapunov–Krasovskii functional method. Delay-dependent sufficient conditions for robust stability of the closed-loop system have been obtained and a design method of robustly stabilizing controllers has been given, these conditions are presented in terms of LMI format. The numeric example has also been given to demonstrate the effectiveness of the new stabilisation conditions obtained for the truck-trailer model with a time-state and input delay and to demonstrate the use and the less conservativeness of the present method.

## References

- [1] T. Takagi and M. Sugeno. Fuzzy identification of its applications to modeling and control. *IEEE Trans. Syst., Man, Cybern.*, vol. SMC-15, no. 1, pp. 116–132, Feb. 1985
- [2] Tanaka K., Ikeda T., Wang H.O. Fuzzy regulators and fuzzy observers: Relaxed stability conditions and LMI-based designs. *IEEE Transactions on Fuzzy Systems*, Vol. 6, No. 2, pp. 250-265, 1998
- [3] Tanaka K., Hua O., Wang H.O., Fuzzy control systems design and analysis a Linear Matrix Inequality, *John Wiley and Sons, New York*, 2001.
- [4] Boyd S., El Ghaoui L., Linear Matrix Inequalities in system and control theory, *Philadelphia, PA: SIAM*, 1994
- [5] J. Yoneyama., Robust stability and stabilization for uncertain Takagi–Sugeno fuzzy time-delay systems. *Fuzzy Sets Syst.*, vol. 158, no. 2,pp. 115–134, Jan. 2007
- [6] X. Jiang and Q.-L. Han. Robust  $H_\infty$  control for uncertain Takagi-Sugeno fuzzy systems with interval time-varying delay. *IEEE Trans. Fuzzy Syst.* vol. 15, no. 2, pp. 321–331, Apr. 2007.
- [7] Chen Peng and all. Sampled-data robust  $H_\infty$  control for T-S fuzzy systems with time delay and uncertainties. *Fuzzy Sets and Systems* 179 (2011) 20 – 33
- [8] Y.Y. Cao, P.M. Frank, Analysis and synthesis of nonlinear time-delay system via fuzzy control approach, *IEEE Transactions on Fuzzy Systems* 8 (2) (2000) 200–211.
- [9] Y. Zhang, A.H. Pheng, Stability of fuzzy systems with bounded uncertain delays, *IEEE Transactions on Fuzzy Systems* 10 (1) (2002) 92–97
- [10] Chen Peng, Dong Yue, and Yu-Chu Tian. New Approach on Robust Delay-Dependent  $H_\infty$  Control for Uncertain T-S Fuzzy Systems With Interval Time-varying delay. *IEEE transactions on fuzzy systems*, vol. 17, no. 4, august 2009
- [12] H. N. Wu and H. X. Li. New approach to delay-dependent stability analysis and stabiliation for continuous time fuzzy systems with time-varying delay. *IEEE Trans. Fuzzy Syst.*, vol. 15, no. 3, pp. 482–493, Jun. 2007.
- [11] P. Chen, Y.C. Tian. Delay-dependent robust stability criteria for uncertain systems with interval time-varying delay. *Journal of Computational and Applied Mathematics* (2007) doi:10.1016/j.cam.2007.03.009.
- [13] Li Li,b, Xiaodong Liu. New results on delay-dependent robust stability criteria of uncertain fuzzy systems with state and input delays. *Information Sciences* 179 (2009) 1134–1148
- [14] L. Xie, Output feedback  $H_1$  control of systems with parameter uncertainty, *International Journal of Control* 63 (4) (1996) 741–750.